

9 – 3 Graph Rational Functions

rational function

$$f(x) = \frac{p(x)}{q(x)}, \text{ where } p \text{ and } q \text{ are polynomial functions}$$

$$\text{EX: } f(x) = \frac{3}{x-2}$$

$$g(x) = \frac{x+2}{x^2-4}$$

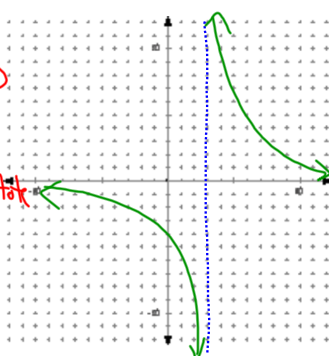
$$h(x) = \frac{x^2+4x-12}{x^2+2x-3}$$

vertical asymptote (infinite discontinuity) – if the rational expression of a function is in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical asymptote. . . .

$$\text{EX: } f(x) = \frac{2}{x-3}$$

$$x \neq 3$$

$x = 3$ Vert. asymptote



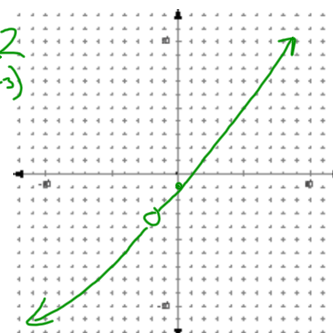
hole (point discontinuity) - if $x - a$ is a factor of the numerator and denominator of a rational function, then there is a hole in the graph at $x = a$.

hole at $x = -2$
(-2, -3)

$$\text{EX: } f(x) = \frac{x^2+x-2}{x+2}$$

$$\frac{(x+2)(x-1)}{(x+2)}$$

$$f(x) = x - 1$$



horizontal asymptote: Given a rational function $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomials:

- a) If the degree of p is less than the degree of q, then $y = 0$ is a horizontal asymptote.
 $N < D$
- b) If the degree of p, with lead coefficient a, is equal to the degree of q, with lead coefficient b, then $y = \frac{a}{b}$ is a horizontal asymptote.
 $N = D$
- c) If the degree of p is greater than the degree of q, then there is **no horizontal asymptote.**
 $N > D$

Examples:

1. $f(x) = \frac{2x}{x+3}$

V.A. $x = -3$

H.A. $y = 2$

2. $f(x) = \frac{x+1}{x^2-1} = \frac{\cancel{x+1}}{(\cancel{x+1})(x-1)} = \frac{1}{x-1}$

Hole $x = -1$ $(-1, -\frac{1}{2})$

V.A. $x = 1$

H.A. $y = 0$

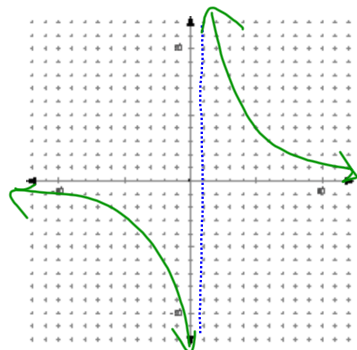
3. $f(x) = \frac{x^2+5x+6}{x+1} = \frac{(x+3)(x+2)}{(x+1)}$

V.A. $x = -1$

$$1. f(x) = \frac{3}{x-1}$$

$$\text{V.A. } x=1$$

$$\text{H.A. } y=0$$

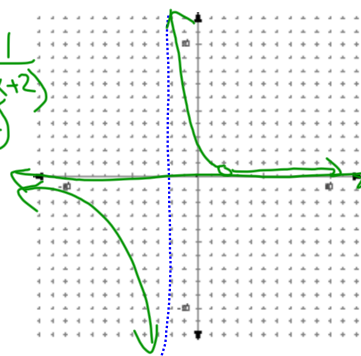


$$2. f(x) = \frac{x-2}{x^2-4} \cdot \frac{1}{(x+2)}$$

$$\text{Hole } (2, \frac{1}{4})$$

$$\text{V.A. } x=-2$$

$$\text{H.A. } y=0$$



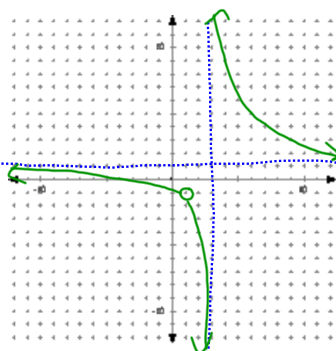
$$3. f(x) = \frac{x^2-1}{x^2-4x+3}$$

$$\frac{3}{2} \cdot \frac{(x+1)(x-1)}{(x-3)(x-1)}$$

$$\text{Hole } (1, -1)$$

$$\text{V.A. } x=3$$

$$\text{H.A. } y=1$$



HW

p489

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23, 25, 31, 33 Graphs